Quarterly Technical Summary Report No. 18 | *April 1 to June 30, 1963*

THE STUDY OF THE ORIGIN AND PROPAGATION OF DISTURBANCES IN THE BURNING OF SOLID PROPELLANTS

Prepared for:

DIRECTOR OF ENGINEERING SCIENCES
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
ATTN: SREP
WASHINGTON 25, D.C.

CONTRACT NO. AF 49(638)-565
ARPA ORDER NO. 317-62, AMEND. NO. 3
PROJECT CODE NO. 9100

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CONTENTS

I INTRODUCTION AND SUMMARY

This report deals with the continued investigation of the reflected-pulse method for measuring the normal acoustic admittance of burning propellant surfaces.

Instrumentation problems proved to be more troublesome than we had anticipated at the close of the previous report period. Although the multi-line recording system is now operational, the noise level which it introduces into the signal is probably too high in view of the accuracy required for the admittance measurements. However, this is a problem of detailed design, and we have made preparations to run about a dozen tests with burning propellant in the near future in order to determine whether the complete instrumentation system operates satisfactorily, at least in principle. A brief discussion of current instrumentation problems is given in Part II of this report.

The need for continued improvements in the instrumentation caused a a lull in experimental activity during the current report period. During this time, progress was made in organizing some of the theoretical results obtained in the course of this contract in a form suitable for publication, and the first of an anticipated series of papers was submitted to the Transactions of the American Mathematical Society, ("Some Theorems Connected with the Lagrange Expansion," by G. M. Muller). A summary of the results of this paper will be found in Part II of this report.

II INSTRUMENTATION

It will be recalled (Quarterly Technical Summary Report No. 16, January 31, 1963) that the multi-line recording system depends on feeding a continuous staircase signal into one channel of a differential amplifier, the microphone signal into the other channel, and a series of unblanking pulses (synchronized with the flat portions of the staircase signal) into the *Z*-input of the oscilloscope (see Fig. 1 of Report No. 16). There is at present about 5% jitter in the amplitude of each step of the staircase signal. This would be of little consequence if the step amplitude could be kept small compared to the maximum amplitude of the microphone signal. However, in order to permit assignment of a given dot in the oscilloscope display to a particular one of several effective base lines, the step amplitude has to be about 20% of the maximum amplitude of the microphone signal. This causes a jitter in the recorded signal of about 1% which is definitely larger than desirable.

The minimum permissible separation of two adjacent base lines is determined by the shortest rise time which occurs in the microphone signal. In order to get a well-resolvable display, we have found it necessary to reduce the upper cutoff frequency to 1.5 kc/s (from the 4 kc/s used in our previous work.) A further reduction is impossible since it would lead to coalescence of incident and reflected pulses.

In assessing these and related difficulties (*e.g.*, the low sensitivity--for our purposes--of available water-cooled microphones) it should be remembered that they are not intrinsic to the reflected-pulse method but derive from the necessity of adapting commercially available instruments to our particular requirements without becoming involved in a major instrument development program.

III THEORY

One of the earliest observations made in our effort to measure admittance by the reflected-pulse method was the occurrence of finite-amplitude effects (see Narrative Progress Report No. 10, June 30, 1961). Eventually, a reasonably complete theory of these effects was developed, and some of the highlights were mentioned in Narrative Progress Report No. 13, March 31, 1962. As is so often the case, it turned out that a significant portion (though by no means all) of this theory had already been discussed in the acoustical literature; however, some of these discussions were rather awkward from a mathematical standpoint and even contained occasional false statements. An attempt to combine the novel portions of our own work with a mathematical critique of the already existing discussions soon revealed the necessity for establishing a number of theorems which really belong to complex-variable theory rather than to applied mathematics or acoustics. It appeared desirable to collect these theorems and their derivations in a separate paper entitled "Some Theorems Connected with the Lagrange Expansion."

In this paper it is first noted that the solution of the initial-value problem for simple waves in the theory of uniaxial, isentropic flow of a perfect fluid can generally be expressed in the form

$$F(\tau, w) = f(z) \tag{1}$$

where z is a solution of the implicit equation

$$z = \tau + w\phi(z) \quad ; \tag{2}$$

here τ is a time variable, w a cartesian space variable, and f and ϕ are functions which depend on the isentropic equation of state of the fluid, the equation of the line on which the initial values are prescribed, and the initial values themselves.

It is next observed that $F(\tau,w)$ as defined by Eqs. (1) and (2) can be expressed as a Lagrange series in the form

$$F(\tau,w) = f(\tau) + \sum_{n=1}^{\infty} \frac{w^n}{n!} \frac{d^{n-1}}{d\tau^{n-1}} \{f'(\tau)[\phi(\tau)]^n\} \quad . \tag{3}$$

Curiously enough, the acoustical literature seems to contain no reference to the Lagrange expansion even though the question of expanding $F(\tau,w)$ as a power series in w has been the subject of many investigations.

The main objective of the paper is to investigate the analytic properties of $F(\tau,w)$ as a function of both independent variables. The principal results are that, under suitable restrictions on ϕ and f, the radius of convergence of the Lagrange expansion is a continuous function of τ, and that within the radius of convergence, $F(\tau,w)$ is an analytic function of both τ and w; moreover, all derivatives of $F(\tau,w)$ (including mixed derivatives) may be obtained by differentiating the Lagrange expansion term by term.

We conclude this report by giving the appropriate form of the Lagrange expansion for the simple problem of finding the pressure-time profile of a low-amplitude one-dimensional pulse as it moves through an inviscid gas of adiabatic index γ. Let $p(t)$ and $\bar{p}(t)$ be the overpressure-time profiles of a pulse at x_1 and x_2, respectively; time is to be counted from the moment of arrival of the pulse at x_1 or x_2, and overpressure measure in units of the ambient pressure. Let c_0 be the ambient sound velocity. Then

$$\bar{p}(t) = \sum_{n=1}^{\infty} \frac{w^{n-1}}{n!} \frac{d^{n-1}}{dt^{n-1}} [p^n(t)] \tag{4}$$

where $w = (x_2 - x_1)(\gamma + 1)/(2\gamma c_0)$. The first two terms of this series are given in Eq. (2) of Narrative Progress Report No. 13.

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